2501/303 2508/303
2502/303 2509/303
2503/303
ENGINEERING MATHEMATICS III
June/July 2022
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN MECHANICAL ENGINEERING
(PRODUCTION OPTION)
(PLANT OPTION)
DIPLOMA IN AUTOMOTIVE ENGINEERING
DIPLOMA IN WELDING AND FABRICATION
DIPLOMA IN CONSTRUCTION PLANT ENGINEERING

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator;
An Abridged Tables of Laplace Transforms and tables of Standard Normal Curves are attached.
*Answer **FIVE** of the following **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given the matrix $A = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 1 & 0 \\ -2 & 1 & -1 \end{bmatrix}$.

Show that $A^2 A^{-1} = A$. (12 marks)

(b) Three forces F_1, F_2 and F_3 in newtons, acting on a mechanical system satisfy the simultaneous equations.

$$2F_1 + F_2 - 2F_3 = -1$$

$$-F_1 + F_2 + 3F_3 = 6$$

$$F_1 - F_2 + 2F_3 = 4$$

Use Cramer's rule to solve the equations. (8 marks)

2. (a) Evaluate the integrals:

(i) $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{1}{\sqrt{1-x^2-y^2}} \, dydx$

(ii) $\int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{\sqrt{x^2+y^2}}^{1-x^2-y^2} y \, dzdxdy$

(12 marks)

(b) Use a double integral to determine the area of the region bounded by the curves

$y = x^2 - 2$ and $y = 6 - x^2$. (8 marks)

3. (a) (i) Show that one root of the equation:

$x^3 + x^2 - 3 = 0$ lies between $x = 1$ and $x = 2$.

(ii) Use the Newton-Raphson method to determine the root in (i), correct to four decimal places.

(11 marks)

(b) Table I represents a cubic polynomial $f(x)$.

Table I

x	-1	0	1	2	3	4
$f(x)$	-4	-1	-2	-1	8	31

Use the Newton-Gregory forward difference interpolation formula to determine the value of $f(0.5)$. (9 marks)

4. (a) Show that the general solution of the differential equation $\frac{x^2}{y}\frac{dy}{dx} = 2x + y$ may be expressed in the form $x^2 + xy = Ay$, where A is an arbitrary constant. (9 marks)

(b) Use the method of undetermined coefficients to solve the differential equation $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = e^{-2t}$, given that when $t = 0$, $x = 0$ and $\frac{dx}{dt} = 2$. (11 marks)

5. (a) From first principles, find the Laplace transform of $f(t) = t\cos 2t$. (8 marks)

(b) A linear time-invariant system is characterized by the differential equation

$2\frac{d^2x}{dt^2} + 7\frac{dx}{dt} + 3x = e^{-t}$. Use Laplace transforms to determine an expression for

$x(t)$, given that when $t = 0$, $x = 1$ and $\frac{dx}{dt} = 0$. (12 marks)

6. (a) Determine the value of the constant a, given that the three vectors $\underset{\sim}{A} = \underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$, $\underset{\sim}{B} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{C} = 2\underset{\sim}{i} + a\underset{\sim}{j} + 6\underset{\sim}{k}$ are coplanar. (5 marks)

(b) Given the vectors $\underset{\sim}{A} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{B} = -\underset{\sim}{i} + 2\underset{\sim}{j} + 4\underset{\sim}{k}$, determine the:

(i) angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$;

(ii) area of the parallelogram spanned by $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

(10 marks)

(c) A vector field $\underset{\sim}{E} = xy\underset{\sim}{i} - yz\underset{\sim}{j} + x^2y\underset{\sim}{k}$ exists in a region of space. Determine the magnitude of $\nabla \times \underset{\sim}{E}$ at the point $(1,2,1)$. (5 marks)

7. (a) (i) Determine the half-range Fourier cosine series of the function

$$f(t)=t^2-\pi t,\ 0<t<\pi.$$

(ii) By setting $t=0$ in the result in (i), show that $\frac{\pi^2}{6}=\sum_{n=1}^{\infty}\frac{1}{n^2}$

(10 marks)

(b) A function $f(t)$ is defined by:

$$f(t)=\begin{cases}2-t, & 0<t<1\\ t, & 1<t<2.\\ f(t+2)\end{cases}$$

(i) Sketch the graph of $f(t)$ in the interval $-1<t<3$, hence;

(ii) determine the Fourier series representation of $f(t)$.

(10 marks)

8. (a) A point $P(x,y)$ moves in such a way that its distance from the point $(0,1)$ is twice its distance from the point $(1,0)$.

(i) Show that the locus of P is a circle;

(i) Determine the centre and radius of the circle.

(7 marks)

(b) Given the scalar field $\phi=xy^2+yz^2$, determine, at the point $(1,-1,1)$:

(i) $\nabla\phi$

(ii) the directional derivative of ϕ in the direction of the vector $\underline{A}=2\underline{i}-\underline{j}+2\underline{k}$;

(iii) $\nabla.\nabla\phi$.

(13 marks)